Vector Identity 12

$$\nabla \cdot \mathbf{x} = 3$$

Proof

$$\begin{aligned}
\nabla \cdot \mathbf{x} &= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i \frac{\partial}{\partial x_i} \right) \cdot \left(\sum_{j=1}^{3} \boldsymbol{\delta}_j x_j \right) \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \cdot \boldsymbol{\delta}_j) \frac{\partial x_j}{\partial x_i} \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \frac{\partial x_j}{\partial x_i} \\
&= \sum_{i=1}^{3} \frac{\partial x_i}{\partial x_i} \\
&= \sum_{i=1}^{3} \delta_{ii} \\
&= \delta_{11} + \delta_{22} + \delta_{33} \\
&= 1 + 1 + 1 \\
&= 3
\end{aligned}$$